Electromagnetic waves and matter in the gravitational field of a black hole

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Abstrakt:
If you compare the circumference path and the diameter path in the vicinity of a black hole, it becomes apparent that the circumference path is shorter and faster than the diameter path. It follows:
Light and matter orbit black holes and cannot reach the event horizon.

Comparison of two possible ways:
According to the QED and the Fermat principle[1], light takes the way of the shortest runtime. Two ways from a Point A to a Point B will be examined. A and B are opposite on a circle with the radius r.

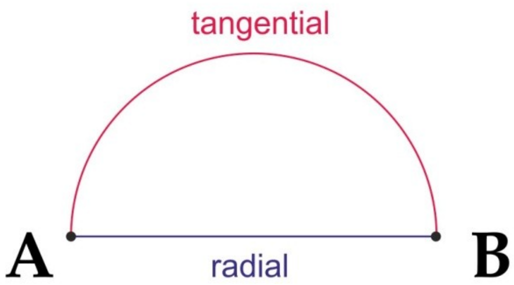

Fig. 1: Light path

Path 1 from A to B is the diameter of the circle:	Path 2 from A to B is the half circumference of the circle:
The light travel time for the diameter path is: $$t_{radial} = \frac{2 * r}{c_{radial}}$$	The light travel time for the circumference path is: $$t_{tangential} = \frac{\pi * r}{c_{tangential}}$$

If the curvature of the room gets in the way, the runtime is extended. The Shapiro delay[2] for the speed of light c is used, for calculate the light travel time.

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[1] https://en.wikipedia.org/wiki/Fermat%27s_principle
[2] See https://en.wikipedia.org/wiki/Shapiro_time_delay or Appendix A

The speed of light for the diameter path is the radial Shapiro speed. $c_{radial} = c * \left(1 - \frac{r_s}{r}\right)$ $r_s = \frac{2 * G * M}{c^2}$ G is the gravitational constant M is the mass of the black hole c is the speed of light r is the distance to the center of the black hole $t_{radial} = \frac{2 * r}{c * (1 - \frac{r_s}{r})}$	The speed of light for the circumference path is the tangential Shapiro speed. $c_{tangential} = c * \sqrt{(1 - \frac{r_s}{r})}$ $r_s = \frac{2 * G * M}{c^2}$ G is the gravitational constant M is the mass of the black hole c is the speed of light r is the distance to the center of the black hole $t_{tangential} = \frac{\pi * r}{c * \sqrt{(1 - \frac{r_s}{r})}}$

If you compare the travel times of the light for both paths graphically, you get the following curves:

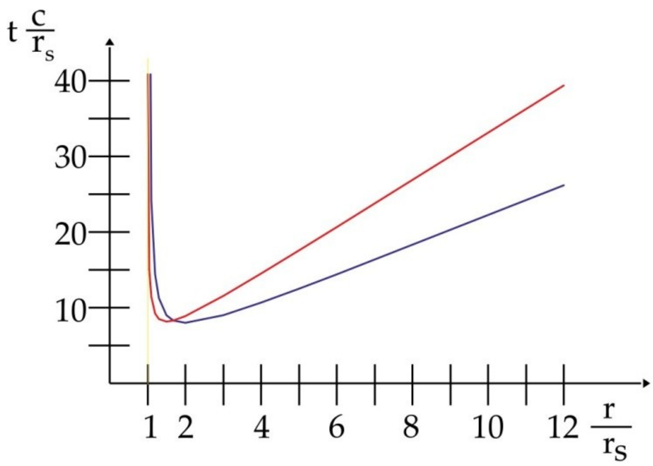

Fig.2: Light travel times; blue: diameter path; red: circumference path

For $\frac{r}{r_s} \leq \frac{1}{1-\frac{4}{\pi^2}} = 1{,}68147$ the travel time of the light for the circumference path (red) is smaller than the travel time for the diameter path (blue). Light chooses the faster way.

With large curvatures, the circumference path is faster than the diameter path. The electromagnetic wave will not be absorbed from the black hole. The wave circles around the black hole on a circular path with the radius $1 < \frac{r}{r_s} \leq 1.68147$. The minimum of the circular path is $\frac{r}{r_s} = 1.5$.

The same applies to matter consisting of an electromagnetic wave. The Poynting vector of the electromagnetic wave will circle the black hole.

Appendix A: Shapiro time delay

From the perspective of a distant observer, the speed of light near a large mass is reduced. A distinction is made between the tangential speed (circular path around the mass) and the radial speed (in the direction of the mass).
Imagine a gravitational funnel. A circular sticker on the funnel surface symbolizes the speeds in the funnel. If you look into the gravity funnel from above, the circular sticker appears oval. The diameter to the center symbolizes the speed to the center (radial speed). The larger diameter symbolizes the tangential speed.

Tangential speed:
With the tangential movement, the curvature of space is constant, because the distance to the mass does not change. The greater the curvature of space, the slower the time passes[3].

$$\Delta t_{tangential} = \Delta t_0 * \sqrt{1 - \frac{r_S}{r}}$$

The observed speed of light decreases in the same ratio:

$$\frac{c_{tangential}}{c} = \frac{\Delta t_{tangential}}{\Delta t_0} = \sqrt{1 - \frac{r_S}{r}}$$

$$c_{tangential} = c\sqrt{1 - \frac{r_S}{r}}$$

Radial speed:
With the radial movement, the time change[4] and the length contraction must be taken into account. The greater the curvature of space, the shorter the lengths and the slower the time passes.

$$\lambda_{radial} = \lambda_0 * \sqrt{1 - \frac{r_S}{r}}$$

$$\Delta t_{radial} = \Delta t_0 * \sqrt{1 - \frac{r_S}{r}}$$

The observed speed of light decreases in the same ratio:

$$\frac{c_{radial}}{c} = \frac{\lambda_{radial}}{\lambda_0} * \frac{\Delta t_{radial}}{\Delta t_0} = 1 - \frac{r_S}{r}$$

$$c_{radial} = c * (1 - \frac{r_S}{r})$$

[3] https://en.wikipedia.org/wiki/Time_dilation#Gravitational_time_dilation
[4] https://en.wikipedia.org/wiki/Redshift#Gravitational_redshift